Solar proton burning, neutrino disintegration of the deuteron and pep process in the relativistic field theory model of the deuteron

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Abstract

The astrophysical factor $S_{\rm pp}(0)$ for the solar proton burning $\rm p + p \to D + e^+ + \nu_e$ is recalculated in the relativistic field theory model of the deuteron (RFMD). We obtain $S_{\rm pp}(0) = 4.08 \times 10^{-25}\,\rm MeV\,b$ which agrees good with the recommended value $S_{\rm pp}(0) = 4.00 \times 10^{-25}\,\rm MeV\,b$. The amplitude of low–energy elastic proton–proton (pp) scattering in the $^1\rm S_0$–state with the Coulomb repulsion contributing to the amplitude of the solar proton burning is described in terms of the S–wave scattering length and the effective range. This takes away the problem pointed out by Bahcall and Kamionkowski (Nucl. Phys. A625 (1997) 893) that in the RFMD one cannot describe low–energy elastic pp scattering with the Coulomb repulsion in agreement with low–energy nuclear phenomenology. The cross section for the neutrino disintegration of the deuteron $\nu_{\rm e} + \rm D \to e^- + p + p$ is calculated with respect to $S_{\rm pp}(0)$ for neutrino energies up to $E_{\nu_{\rm e}} \le 10\,\rm MeV$. The results can be used for the analysis of the data which will be obtained in the experiments planned by SNO. The astrophysical factor $S_{\rm pep}(0)$ for the process $\rm p + e^- + p \to \nu_e + D$ (or pep–process) is calculated relative to $S_{\rm pp}(0)$ in complete agreement with the result obtained by Bahcall and May (ApJ. 155 (1969) 501).

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1 Introduction

The weak nuclear process $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\mathrm{e}}$, the solar proton burning or proton–proton (pp) fusion, plays an important role in Astrophysics [1,2]. It gives start for the p–p chain of nucleosynthesis in the Sun and the main–sequence stars [1,2]. In the Standard Solar Model (SSM) [3] the total (or bolometric) luminosity of the Sun $L_{\odot} = (3.846 \pm 0.008) \times 10^{26}\,\mathrm{W}$ is normalized to the astrophysical factor $S_{\mathrm{pp}}(0)$ for pp fusion. The recommended value $S_{\mathrm{pp}}(0) = 4.00 \times 10^{-25}\,\mathrm{MeVb}$ [4] has been found by averaging over the results obtained in the Potential model approach (PMA) [5,6] and the Effective Field Theory (EFT) approach [7,8]. However, as has been shown recently in Ref.[9] *the inverse and forward helioseismic approach indicate the higher values of* $S_{\mathrm{pp}}(0)$ *seem more favoured*, for example, $S_{\mathrm{pp}}(0) = 4.20 \times 10^{-25}\,\mathrm{MeVb}$ and higher [9]. Of course, accounting for the experimental errors the recommended value does not contradict the result obtained in Ref.[9].

In Refs.[10-13] we have developed a relativistic field theory model of the deuteron (RFMD). In turn, in Ref.[14] we have suggested a modified version of the RFMD which is not well defined due to a violation of Lorentz invariance of the effective four–nucleon interaction describing $\mathrm{N} + \mathrm{N} \to \mathrm{N} + \mathrm{N}$ transitions. This violation has turned out to be incompatible with a dominance of one–nucleon loop anomalies which are Lorentz covariant. Thereby, the astrophysical factor $S_{\mathrm{pp}}(0)$ calculated in the modified version of the RFMD [14] and enhanced by a factor of 1.4 with respect to the recommended value [4] is not good established. This result demands the confirmation within the original RFMD [10–13] by using the technique expounded in Ref.[13].

As has been shown in Ref. [12] the RFMD is motivated by QCD. The deuteron appears in the nuclear phase of QCD as a neutron–proton collective excitation – a Cooper np–pair induced by a phenomenological local four–nucleon interaction. Strong low–energy interactions of the deuteron coupled to itself and other particles are described in terms of one–nucleon loop exchanges. The one–nucleon loop exchanges allow to transfer nuclear flavours from an initial to a final nuclear state by a minimal way and to take into account contributions of nucleon–loop anomalies determined completely by one–nucleon loop diagrams. The dominance of contributions of nucleon–loop anomalies has been justified in the large N_C expansion, where N_C is the number of quark colours [13]. Unlike the PMA and the EFT approach the RFMD takes into account non–perturbative contributions of high–energy (short–distance) fluctuations of virtual nucleon (N) and anti–nucleon ($\bar{N}$) fields, $N\bar{N}$ fluctuations, in the form of one–nucleon loop anomalies. In accord the analysis carried out in Refs.[15] nucleon–loop anomalies can be interpreted as non–perturbative contributions of the nucleon Dirac sea. The description of one–nucleon loop anomalies goes beyond the scope of both the PMA and the EFT approach due to the absence in these approaches anti–nucleon degrees of freedom related to the nucleon Dirac sea. However, one should notice that in low–energy nuclear physics the nucleon Dirac sea cannot be ignored fully [16]. For example, high–energy $N\bar{N}$ fluctuations of the nucleon Dirac sea polarized by the nuclear medium decrease the scalar nuclear density in the nuclear interior of finite nuclei by 15% [16]. This effect has been obtained within quantum field theoretic approaches in terms of one–nucleon loop exchanges.

In this paper we revise the value of $S_{\mathrm{pp}}(0)$ obtained in Ref. [14]. For this aim we apply the technique developed in the RFMD [13] for the description of contributions of low–energy elastic nucleon–nucleon scattering in the ${}^1\mathrm{S}_0$–state to amplitudes of electromagnetic

and weak nuclear processes. This technique implies the summation of an infinite series of one–nucleon loop diagrams and the evaluation of the result of the summation in leading order in large N_C expansion [13]. The application of this method to the evaluation of the cross sections for the anti–neutrino disintegration of the deuteron induced by charged $\bar{\nu}_{\rm e}$ + D $\to$ $\rm e^+$ + n + n and neutral $\bar{\nu}_{\rm e}$ + D $\to$ $\bar{\nu}_{\rm e}$ + n + p weak currents gave the results agreeing good with the experimental data. The reaction $\bar{\nu}_{\rm e}$ + D $\to$ $\rm e^+$ + n + n is, in the sense of charge independence of weak interaction strength, equivalent to the reaction p + p $\to$ D + $\rm e^+$ + $\nu_{\rm e}$. Therefore, the application of the same technique to the description of the reaction p + p $\to$ D + $\rm e^+$ + $\nu_{\rm e}$ should give a result of a good confidence level.

The paper is organized as follows. In Sect. 2 we evaluate the amplitude of the solar proton burning. We show that the contribution of low–energy elastic pp scattering in the $^1\rm S_0$–state with the Coulomb repulsion is described in agreement with low–energy nuclear phenomenology in terms of the S–wave scattering length and the effective range. This takes away the problem pointed out by Bahcall and Kamionkowski [17] that in the RFMD one cannot describe low–energy elastic pp scattering with the Coulomb repulsion in agreement with low–energy nuclear phenomenology. In Sect. 3 we evaluate the astrophysical factor for the solar proton burning and obtain the value $S_{\rm pp}(0) = 4.08\times 10^{-25}\,{\rm MeV\,b}$ agreeing good with the recommended one $S_{\rm pp}(0) = 4.00\times 10^{-25}\,{\rm MeV\,b}$. In Sect. 4 we evaluate the cross section for the neutrino disintegration of the deuteron $\nu_{\rm e}$ + D $\to$ $\rm e^-$ + p + p caused by the charged weak current with respect to $S_{\rm pp}(0)$. In Sect. 5 we adduce the evaluation of the astrophysical factor $S_{\rm pep}(0)$ of the reaction p + $\rm e^-$ + p $\to$ D + $\nu_{\rm e}$ or pep–process relative to $S_{\rm pp}(0)$. In the Conclusion we discuss the obtained results.

2 Amplitude of solar proton burning and low–energy elastic proton–proton scattering

For the description of low–energy transitions N + N $\to$ N + N in the reactions n + p $\to$ D + γ, γ + D $\to$ n + p, $\bar{\nu}_{\rm e}$ + D $\to$ $\rm e^+$ + n + n and p + p $\to$ D + $\rm e^+$ + $\nu_{\rm e}$, where nucleons are in the $^1\rm S_0$–state, we apply the effective local four–nucleon interactions [11–13]:

$$\begin{aligned}
&{\cal L}^{\rm NN\to NN}_{\rm eff}(x) = G_{\pi\rm NN}\,\{[\bar{n}(x)\gamma_{\mu}\gamma^5 p^c(x)][\bar{p^c}(x)\gamma^{\mu}\gamma^5 n(x)]\\
&+\frac{1}{2}\,[\bar{n}(x)\gamma_{\mu}\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^{\mu}\gamma^5 n(x)] + \frac{1}{2}\,[\bar{p}(x)\gamma_{\mu}\gamma^5 p^c(x)][\bar{p^c}(x)\gamma^{\mu}\gamma^5 p(x)]\\
&+(\gamma_{\mu}\gamma^5\otimes\gamma^{\mu}\gamma^5 \to \gamma^5\otimes\gamma^5)\},
\end{aligned}\tag{2.1}$$

where $n(x)$ and $p(x)$ are the operators of the neutron and the proton interpolating fields, $n^c(x) = C\bar{n}^T(x)$ and so on, then C is a charge conjugation matrix and T is a transposition. The effective coupling constant $G_{\pi\rm NN}$ is defined by [11–13]

$$G_{\pi\rm NN} = \frac{g^2_{\pi\rm NN}}{4M^2_{\pi}} - \frac{2\pi a_{\rm np}}{M_{\rm N}} = 3.27\times 10^{-3}\,{\rm MeV}^{-2},\tag{2.2}$$

where $g_{\pi\rm NN} = 13.4$ is the coupling constant of the πNN interaction, $M_{\pi} = 135\,{\rm MeV}$ is the pion mass, $M_{\rm p} = M_{\rm n} = M_{\rm N} = 940\,{\rm MeV}$ is the mass of the proton and the neutron neglecting the electromagnetic mass difference, which is taken into account only for the calculation of the phase volumes of the final states of the reactions p + p $\to$ D + $\rm e^+$ +

$\nu_{\rm e}$, $\nu_{\rm e}$ + D → $\rm e^-$ + p + p and p + $\rm e^-$ + p → D + $\nu_{\rm e}$, and $a_{\rm np} = (-23.75 \pm 0.01)\,{\rm fm}$ is the S–wave scattering length of np scattering in the ${}^1{\rm S}_0$–state.

The effective Lagrangian for the low–energy nuclear transition p + p → D + $\rm e^+$ + $\nu_{\rm e}$ has been calculated in Ref. [12] and reads

$$\mathcal{L}_{\rm pp\to De^+\nu_e}(x) = -ig_{\rm A}G_{\pi\rm NN}M_{\rm N}\frac{G_{\rm V}}{\sqrt{2}}\frac{3g_{\rm V}}{4\pi^2}\,D^{\dagger}_{\mu}(x)\,[\bar{p^c}(x)\gamma^5 p(x)]\,[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^{\mu}(1-\gamma^5)\psi_{\rm e}(x)]. \quad (2.3)$$

where $G_{\rm V} = G_{\rm F}\cos\vartheta_C$ with $G_{\rm F} = 1.166\times 10^{-11}\,{\rm MeV}^{-2}$ and ϑ_C are the Fermi weak coupling constant and the Cabibbo angle $\cos\vartheta_C = 0.975$, $g_{\rm A} = 1.2670\pm 0.0035$ [18] and $g_{\rm V}$ is a phenomenological coupling constant of the RFMD related to the electric quadrupole moment of the deuteron $Q_{\rm D} = 0.286\,{\rm fm}^2$ [11]: $g^2_{\rm V} = 2\pi^2 Q_{\rm D}M^2_{\rm N}$. Then, $D_{\mu}(x)$, $\psi_{\nu_{\rm e}}(x)$ $\psi_{\rm e}(x)$ are the interpolating fields of the deuteron and leptonic pair, respectively.

The effective Lagrangian Eq.(2.3) defines the effective vertex of the low–energy nuclear transition p + p → D + $\rm e^+$ + $\nu_{\rm e}$

$$\begin{aligned} i\mathcal{M}({\rm p + p \to D + e^+} + \nu_e) &= G_{\rm V}\,g_{\rm A}M_{\rm N}\,G_{\pi\rm NN}\,\frac{3g_{\rm V}}{4\pi^2}\,e^*_{\mu}(k_{\rm D})\,[\bar{u}(k_{\nu_{\rm e}})\gamma^{\mu}(1-\gamma^5)v(k_{\rm e^+})] \\ &\quad\times[\bar{u^c}(p_2)\gamma^5 u(p_1)], \end{aligned} \quad (2.4)$$

where $e^*_{\mu}(k_{\rm D})$ is a 4–vector of a polarization of the deuteron, $u(k_{\nu_{\rm e}})$, $v(k_{\rm e^+})$, $u(p_2)$ and $u(p_1)$ are the Dirac bispinors of neutrino, positron and two protons, respectively.

In order to evaluate the contribution of low–energy elastic pp scattering we have to determine the effective vertex of the p + p → p + p transition accounting for the Coulomb repulsion between the protons. For this aim we suggest to use the effective local four–nucleon interaction Eq.(2.1) and take into account the Coulomb repulsion in terms of the explicit Coulomb wave function of the protons. This yields

$$V_{\rm pp\to pp}(k',k) = G_{\pi\rm NN}\,\psi^*_{\rm pp}(k'\,)\,[\bar{u}(p'_2)\gamma^5 u^c(p'_1)]\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,\psi_{\rm pp}(k), \quad (2.5)$$

where $\psi_{\rm pp}(k)$ and $\psi^*_{\rm pp}(k'\,)$ are the explicit Coulomb wave functions of the relative movement of the protons taken at zero relative radius, and k and k' are relative 3–momenta of the protons $\vec{k} = (\vec{p}_1 - \vec{p}_2)/2$ and $\vec{k}^{\,\prime} = (\vec{p}^{\,\prime}_1 - \vec{p}^{\,\prime}_2)/2$ in the initial and final states. The explicit form of $\psi_{\rm pp}(k)$ we take following Kong and Ravndal [8] (see also [19])

$$\psi_{\rm pp}(k) = e^{-\pi/4kr_C}\,\Gamma\Bigg(1+\frac{i}{2kr_C}\Bigg), \quad (2.6)$$

where $r_C = 1/M_{\rm N}\alpha = 28.82\,{\rm fm}$ and $\alpha = 1/137$ are the Bohr radius of a proton and the fine structure constant. The squared value of the modulo of $\psi_{\rm pp}(k)$ is given by

$$|\psi_{\rm pp}(k)|^2 = C^2_0(k) = \frac{\pi}{kr_C}\,\frac{1}{e^{\pi/kr_C}-1}, \quad (2.7)$$

where $C_0(k)$ is the Gamow penetration factor [1,2,19]. We would like to emphasize that the wave function Eq.(2.6) is defined only by a regular solution of the Schrödinger equation for the pure Coulomb potential [19].

By taking into account the contribution of the Coulomb wave function and summing up an infinite series of one–proton loop diagrams the amplitude of the solar proton burning can be written in the form

$$i\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+\mathrm{e}^+ + \nu_e) = G_{\mathrm{V}}\, g_{\mathrm{A}} M_{\mathrm{N}}\, G_{\pi\mathrm{NN}}\,\frac{3 g_{\mathrm{V}}}{4\pi^2}\, e^*_{\mu}(k_{\mathrm{D}})\,[\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^{\mu}(1-\gamma^5)v(k_{\mathrm{e}^+})]\,\mathcal{F}^{\mathrm{e}}_{\mathrm{pp}}$$
$$\times\frac{[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,\psi_{\mathrm{pp}}(k)}{1+\dfrac{G_{\pi\mathrm{NN}}}{16\pi^2}\displaystyle\int\frac{d^4p}{\pi^2 i}\,|\psi_{\mathrm{pp}}(|\vec{p}+\vec{Q}\,|)|^2\mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{P}-\hat{Q}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{Q}}\Big\}}. \tag{2.8}$$

where $P = p_1 + p_2 = (2\sqrt{k^2+M^2_{\mathrm{N}}},\vec{0}\,)$ is the 4–momentum of the pp–pair in the center of mass frame; $Q = a\,P + b\,K = a\,(p_1+p_2) + b\,(p_1-p_2)$ is an arbitrary shift of virtual momentum with arbitrary parameters a and b, and in the center of mass frame $K = p_1 - p_2 = (0, 2\,\vec{k}\,)$ [14]. The parameters a and b can be functions of k. The factor $\mathcal{F}^{\mathrm{e}}_{\mathrm{pp}}$ describes the overlap of the Coulomb and strong interactions [10]. It is analogous the overlap integral in the PMA [5]. We calculate this factor below.

The evaluation of the momentum integral runs the way expounded in [14]. Keep only leading contributions in the large N_C expansion [13,14] we obtain

$$\begin{aligned}&\int\frac{d^4p}{\pi^2 i}\,|\psi_{\mathrm{pp}}(|\vec{p}+\vec{Q}\,|)|^2\mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{P}-\hat{Q}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{Q}}\Big\} =\\ &= -8\,a\,(a+1)\,M^2_{\mathrm{N}} + 8\,(b^2 - a\,(a+1))\,k^2 - i\,8\pi\,M_{\mathrm{N}}\,k\,|\psi_{\mathrm{pp}}(k)|^2 =\\ &= -8\,a\,(a+1)\,M^2_{\mathrm{N}} + 8\,(b^2 - a\,(a+1))\,k^2 - i\,8\pi\,M_{\mathrm{N}}\,k\,C^2_0(k).\end{aligned} \tag{2.9}$$

Substituting Eq.(2.9) in Eq.(2.8) we get

$$\begin{aligned}&i\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+\mathrm{e}^+ + \nu_e) = G_{\mathrm{V}}\, g_{\mathrm{A}} M_{\mathrm{N}}\, G_{\pi\mathrm{NN}}\,\frac{3 g_{\mathrm{V}}}{4\pi^2}\,\mathcal{F}^{\mathrm{e}}_{\mathrm{pp}}\\ &\times e^*_{\mu}(k_{\mathrm{D}})\,[\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^{\mu}(1-\gamma^5)v(k_{\mathrm{e}^+})]\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,e^{-\pi/4kr_C}\,\Gamma\Big(1+\frac{i}{2kr_C}\Big)\\ &\Big[1 - a(a+1)\frac{G_{\pi\mathrm{NN}}}{2\pi^2}\,M^2_{\mathrm{N}} + \frac{G_{\pi\mathrm{NN}}}{2\pi^2}\,(b^2 - a\,(a+1))\,k^2 - i\,\frac{G_{\pi\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,k\,C^2_0(k)\Big]^{-1}.\end{aligned} \tag{2.10}$$

In order to reconcile the contribution of low–energy elastic pp scattering with low–energy nuclear phenomenology [19] we should make a few changes. For this aim we should rewrite Eq.(2.10) in more convenient form

$$\begin{aligned}&i\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+\mathrm{e}^+ + \nu_e) = G_{\mathrm{V}}\, g_{\mathrm{A}} M_{\mathrm{N}}\, G_{\pi\mathrm{NN}}\,\frac{3 g_{\mathrm{V}}}{4\pi^2}\,\mathcal{F}^{\mathrm{e}}_{\mathrm{pp}}\\ &\times e^*_{\mu}(k_{\mathrm{D}})\,[\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^{\mu}(1-\gamma^5)v(k_{\mathrm{e}^+})]\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,e^{i\sigma_0(k)}\,C_0(k)\\ &\Big[1 - a(a+1)\frac{G_{\pi\mathrm{NN}}}{2\pi^2}\,M^2_{\mathrm{N}} + \frac{G_{\pi\mathrm{NN}}}{2\pi^2}\,(b^2 - a\,(a+1))\,k^2 - i\,\frac{G_{\pi\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,k\,C^2_0(k)\Big]^{-1}.\end{aligned} \tag{2.11}$$

We have denoted

$$e^{-\pi/4kr_C}\,\Gamma\Big(1+\frac{i}{2kr_C}\Big) = e^{i\sigma_0(k)}\,C_0(k)\ ,\ \ \sigma_0(k) \;=\; \arg\Gamma\Big(1+\frac{i}{2kr_C}\Big), \tag{2.12}$$

where $\sigma_0(k)$ is a pure Coulomb phase shift.

Now, let us rewrite the denominator of the amplitude Eq.(2.11) in the equivalent form

$$\Big\{\cos\sigma_0(k)\Big[1 - a(a+1)\frac{G_{\pi\rm NN}}{2\pi^2}\,M^2_{\rm N} + \frac{G_{\pi\rm NN}}{2\pi^2}\,(b^2 - a\,(a+1))\,k^2\Big]$$
$$-\sin\sigma_0(k)\,\frac{G_{\pi\rm NN}M_{\rm N}}{2\pi}\,k\,C^2_0(k)\Big\} - i\,\Big\{\cos\sigma_0(k)\,\frac{G_{\pi\rm NN}M_{\rm N}}{2\pi}\,k\,C^2_0(k)$$
$$+\sin\sigma_0(k)\Big[1 - a(a+1)\frac{G_{\pi\rm NN}}{2\pi^2}\,M^2_{\rm N} + \frac{G_{\pi\rm NN}}{2\pi^2}\,(b^2 - a\,(a+1))\,k^2\Big]\Big\} =$$
$$= \frac{1}{Z}\Big[1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C) + i\,a^{\rm e}_{\rm pp}\,k\,C^2_0(k)\Big], \tag{2.13}$$

where we have denoted

$$\frac{1}{Z}\Big[1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C)\Big] = -\sin\sigma_0(k)\,\frac{G_{\pi\rm NN}M_{\rm N}}{2\pi}\,k\,C^2_0(k)$$
$$+\cos\sigma_0(k)\Big[1 - a(a+1)\frac{G_{\pi\rm NN}}{2\pi^2}\,M^2_{\rm N} + \frac{G_{\pi\rm NN}}{2\pi^2}\,(b^2 - a\,(a+1))\,k^2\Big],$$
$$-\frac{1}{Z}\,a^{\rm e}_{\rm pp}\,k\,C^2_0(k) = \cos\sigma_0(k)\frac{G_{\pi\rm NN}M_{\rm N}}{2\pi}\,k\,C^2_0(k)$$
$$+\sin\sigma_0(k)\Big[1 - a(a+1)\frac{G_{\pi\rm NN}}{2\pi^2}\,M^2_{\rm N} + \frac{G_{\pi\rm NN}}{2\pi^2}\,(b^2 - a\,(a+1))\,k^2\Big]. \tag{2.14}$$

Here Z is a constant which will be removed the renormalization of the wave functions of the protons, $a^{\rm e}_{\rm pp} = (-7.8196 \pm 0.0026)\,{\rm fm}$ and $r^{\rm e}_{\rm pp} = 2.790 \pm 0.014\,{\rm fm}$ [20] are the S–wave scattering length and the effective range of pp scattering in the ${}^1{\rm S}_0$–state with the Coulomb repulsion, and $h(2kr_C)$ is defined by [19]

$$h(2kr_C) = -\gamma + \ell n(2kr_C) + \sum^{\infty}_{n=1}\frac{1}{n(1 + 4n^2k^2r^2_C)}. \tag{2.15}$$

The validity of the relations Eq.(2.14) assumes the dependence of parameters a and b on the relative momentum k.

After the changes Eq.(2.11)–Eq.(2.14) the amplitude Eq.(2.10) takes the form

$$i{\cal M}({\rm p} + {\rm p} \to {\rm D} + {\rm e}^+ + \nu_e) = G_{\rm V}\,g_{\rm A}M_{\rm N}\,G_{\pi\rm NN}\,\frac{3g_{\rm V}}{4\pi^2}\,e^*_{\mu}(k_{\rm D})\,[\bar{u}(k_{\nu_{\rm e}})\gamma^{\mu}(1 - \gamma^5)v(k_{{\rm e}^+})]\,{\cal F}^{\rm e}_{\rm pp}$$
$$\times\,\frac{C_0(k)}{1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C) + i\,a^{\rm e}_{\rm pp}\,k\,C^2_0(k)}\,Z\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]. \tag{2.16}$$

Following [14] and renormalizing the wave functions of the protons $\sqrt{Z}u(p_2) \to u(p_2)$ and $\sqrt{Z}u(p_1) \to u(p_1)$ we obtain the amplitude of the solar proton burning

$$i{\cal M}({\rm p} + {\rm p} \to {\rm D} + {\rm e}^+ + \nu_e) = G_{\rm V}\,g_{\rm A}M_{\rm N}\,G_{\pi\rm NN}\,\frac{3g_{\rm V}}{4\pi^2}\,e^*_{\mu}(k_{\rm D})\,[\bar{u}(k_{\nu_{\rm e}})\gamma^{\mu}(1 - \gamma^5)v(k_{{\rm e}^+})]\,{\cal F}^{\rm e}_{\rm pp}$$
$$\times\,\frac{C_0(k)}{1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C) + i\,a^{\rm e}_{\rm pp}\,k\,C^2_0(k)}\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,F_{\rm D}(k^2), \tag{2.17}$$

where we have introduced too an universal form factor [14]

$$F_{\rm D}(k^2) = \frac{1}{1 + r^2_{\rm D}k^2} \tag{2.18}$$

describing a spatial smearing of the deuteron coupled to the NN system in the ${}^1{\rm S}_0$–state at low energies; $r_{\rm D} = 1/\sqrt{\varepsilon_{\rm D}M_{\rm N}} = 4.315\,{\rm fm}$ is the radius of the deuteron and $\varepsilon_{\rm D} = 2.225\,{\rm MeV}$ is the binding energy of the deuteron.

The real part of the denominator of the amplitude Eq.(2.17) is in complete agreement with a phenomenological relation [19]

$${\rm ctg}\delta^{\rm e}_{\rm pp}(k) = \frac{1}{C^2_0(k)\,k}\Big[-\frac{1}{a^{\rm e}_{\rm pp}} + \frac{1}{2}\,r^{\rm e}_{\rm pp}k^2 - \frac{1}{r_{\rm C}}\,h(2kr_{\rm C})\Big], \tag{2.19}$$

describing the phase shift $\delta^{\rm e}_{\rm pp}(k)$ of low–energy elastic pp scattering in terms of the S–wave scattering length $a^{\rm e}_{\rm pp}$ and the effective range $r^{\rm e}_{\rm pp}$. As has been pointed out [19] the expansion Eq.(2.19) is valid up to $T_{\rm pp} \le 10\,{\rm MeV}$, where $T_{\rm pp} = k^2/M_{\rm N}$ is a kinetic energy of the relative movement of the protons.

Thus, we argue that the contribution of low–energy elastic pp scattering to the amplitude of the solar proton burning is described in agreement with low–energy nuclear phenomenology in terms of the S–wave scattering length $a^{\rm e}_{\rm pp}$ and the effective range $r^{\rm e}_{\rm pp}$ taken from the experimental data [20]. This takes away the problem pointed out by Bahcall and Kamionkowski [17] that in the RFMD with the local four–nucleon interaction given by Eq.(2.1) one cannot describe low–energy elastic pp scattering with the Coulomb repulsion in agreement with low–energy nuclear phenomenology.

Now let us proceed to the evaluation of ${\cal F}^{\rm e}_{\rm pp}$. For this aim we should write down the matrix element of the transition ${\rm p} + {\rm p} \to {\rm D} + {\rm e}^+ + \nu_{\rm e}$ with the Coulomb repulsion. The required matrix element has been derived in Refs.[11,14] and reads

$$\begin{aligned}
&i{\cal M}_C({\rm p} + {\rm p} \to {\rm D} + {\rm e}^+ + \nu_e) = \\
&= G_{\rm V}\,g_{\rm A}M_{\rm N}\,G_{\pi{\rm NN}}\,\frac{3g_{\rm V}}{4\pi^2}\,C_0(k)\,[\bar{u}(k_{\nu_{\rm e}})\gamma^{\mu}(1-\gamma^5)v(k_{{\rm e}^+})]\,e^*_{\mu}(k_{\rm D})\\
&\times\{-[\bar{u^c}(p_2)\gamma_{\alpha}\gamma^5u(p_1)]{\cal J}^{\alpha\mu\nu}_C(k_{\rm D},k_{\ell}) - [\bar{u^c}(p_2)\gamma^5u(p_1)]\,{\cal J}^{\mu\nu}_C(k_{\rm D},k_{\ell})\},
\end{aligned}\tag{2.20}$$

where $k_{\rm D}$ and k_{ℓ} are 4–momenta of the deuteron and the leptonic pair, respectively. The structure functions ${\cal J}^{\alpha\mu\nu}(k_{\rm D},k_{\ell})$ and ${\cal J}^{\mu\nu}(k_{\rm D},k_{\ell})$ are determined by [11,14]

$$\begin{aligned}
&{\cal J}^{\alpha\mu\nu}_C(k_{\rm D},k_{\ell}) = \int\frac{d^4p}{\pi^2i}\,e^{-\pi/4|\vec{q}\,|r_C}\,\Gamma\Big(1 - \frac{i}{2|\vec{q}\,|r_C}\Big)\\
&\times{\rm tr}\Big\{\gamma^{\alpha}\gamma^5\frac{1}{M_{\rm N} - \hat{p} + \hat{k}_{\rm D}}\gamma^{\mu}\frac{1}{M_{\rm N} - \hat{p}}\gamma^{\nu}\gamma^5\frac{1}{M_{\rm N} - \hat{p} - \hat{k}_{\ell}}\Big\},\\
&{\cal J}^{\mu\nu}_C(k_{\rm D},k_{\ell}) = \int\frac{d^4p}{\pi^2i}\,e^{-\pi/4|\vec{q}\,|r_C}\,\Gamma\Big(1 - \frac{i}{2|\vec{q}\,|r_C}\Big)\\
&\times{\rm tr}\Big\{\gamma^5\frac{1}{M_{\rm N} - \hat{p} + \hat{k}_{\rm D}}\gamma^{\mu}\frac{1}{M_{\rm N} - \hat{p}}\gamma^{\nu}\gamma^5\frac{1}{M_{\rm N} - \hat{p} - \hat{k}_{\ell}}\Big\},
\end{aligned}\tag{2.21}$$

where $\vec{q} = \vec{p} + (\vec{k}_{\ell} - \vec{k}_{\rm D})/2$.

For the subsequent analysis it is convenient to represent the structure functions in the form of two terms

$$
\begin{aligned}
\mathcal{J}^{\alpha\mu\nu}_{C}(k_{\rm D},k_\ell) &= \mathcal{J}^{\alpha\mu\nu}_{SS}(k_{\rm D},k_\ell)+\mathcal{J}^{\alpha\mu\nu}_{SC}(k_{\rm D},k_\ell),\\
\mathcal{J}^{\mu\nu}_{C}(k_{\rm D},k_\ell) &= \mathcal{J}^{\mu\nu}_{SS}(k_{\rm D},k_\ell)+\mathcal{J}^{\mu\nu}_{SC}(k_{\rm D},k_\ell).
\end{aligned}\tag{2.22}
$$

The decomposition is caused by the change

$$
e^{-\pi/4|\vec{q}\,|r_C}\,\Gamma\Big(1-\frac{i}{2|\vec{q}\,|r_C}\Big)=1+\Big[e^{-\pi/4|\vec{q}\,|r_C}\,\Gamma\Big(1-\frac{i}{2|\vec{q}\,|r_C}\Big)-1\Big],\tag{2.23}
$$

where the first term gives the contribution to the SS part of the structure functions defined by strong interactions only, while the second one vanishes at $r_C\to\infty$ (or $\alpha\to 0$) and describes the contribution to the SC part of the structure functions caused by both strong and Coulomb interactions.

The procedure of the evaluation of the structure functions Eq.(2.21) and Eq.(2.22) has been described in detail in Ref.[11,14]. Following this procedure we obtain $\mathcal{F}^{\rm e}_{\rm pp}$ in the form

$$
\begin{aligned}
&\mathcal{F}^{\rm e}_{\rm pp}=\\
&=1+\frac{32}{9}\int\limits^{\infty}_{0}dp\,p^2\,\Big[e^{-\pi/4pr_C}\,\Gamma\Big(1-\frac{i}{2pr_C}\Big)-1\Big]\Big[\frac{M^2_{\rm N}}{(M^2_{\rm N}+p^2)^{5/2}}-\frac{7}{16}\,\frac{1}{(M^2_{\rm N}+p^2)^{3/2}}\Big]\\
&=1+\frac{32}{9}\int\limits^{\infty}_{0}dv\,v^2\,\Big[e^{-\alpha\pi/4v}\,\Gamma\Big(1-\frac{i\alpha}{2v}\Big)-1\Big]\Big[\frac{1}{(1+v^2)^{5/2}}-\frac{7}{16}\,\frac{1}{(1+v^2)^{3/2}}\Big].
\end{aligned}\tag{2.24}
$$

The integral can be estimated perturbatively. The result reads

$$
\mathcal{F}^{\rm e}_{\rm pp}=1+\alpha\,\Big(\frac{5\pi}{54}-i\,\frac{5\gamma}{27}\Big)+O(\alpha^2).\tag{2.25}
$$

The numerical value of $|\mathcal{F}^{\rm e}_{\rm pp}|^2$ is

$$
|\mathcal{F}^{\rm e}_{\rm pp}|^2=1+\alpha\,\frac{5\pi}{27}+O(\alpha^2)=1+(4.25\times 10^{-3})\simeq 1.\tag{2.26}
$$

The contribution of the Coulomb field Eq.(2.26) inside the one–nucleon loop diagrams is found small. This is because of the integrals are concentrated around virtual momenta of order of $M_{\rm N}$ which is of order $M_{\rm N}\sim N_C$ in the large N_C expansion [12]. For the calculation of the astrophysical factor $S_{\rm pp}(0)$ we can set $\mathcal{F}^{\rm e}_{\rm pp}=1$.

3 Astrophysical factor for solar proton burning

The amplitude Eq.(2.17) squared, averaged over polarizations of protons and summed over polarizations of final particles reads

$$
\overline{|\mathcal{M}(\mathrm{p}+\mathrm{p}\to\mathrm{D}+\mathrm{e}^{+}+\nu_e)|^2}=G^2_{\rm V}g^2_{\rm A}M^4_{\rm N}\,G^2_{\pi\rm NN}\,\frac{9Q_{\rm D}}{8\pi^2}\,F^2_{\rm D}(k^2)
$$

$$\times \frac{C_0^2(k)}{\Big[1 - \frac{1}{2}\,a^{\rm e}_{\rm pp} r^{\rm e}_{\rm pp} k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C)\Big]^2 + (a^{\rm e}_{\rm pp})^2 k^2 C^4_0(k)}\Bigg(- g^{\alpha\beta} + \frac{k^{\alpha}_{\rm D} k^{\beta}_{\rm D}}{M^2_{\rm D}}\Bigg)$$

$$\times {\rm tr}\{(-m_{\rm e} + \hat{k}_{\rm e^+})\gamma_{\alpha}(1-\gamma^5)\hat{k}_{\nu_{\rm e}}\gamma_{\beta}(1-\gamma^5)\} \times \frac{1}{4} \times {\rm tr}\{(M_{\rm N} - \hat{p}_2)\gamma^5 (M_{\rm N} + \hat{p}_1)\gamma^5\}, \quad (3.1)$$

where $m_{\rm e} = 0.511\,{\rm MeV}$ is the positron mass, and we have used the relation $g^2_{\rm V} = 2\,\pi^2\,Q_{\rm D}\,M^2_{\rm N}$.

In the low–energy limit the computation of the traces yields

$$\Bigg(- g^{\alpha\beta} + \frac{k^{\alpha}_{\rm D} k^{\beta}_{\rm D}}{M^2_{\rm D}}\Bigg) \times {\rm tr}\{(-m_{\rm e} + \hat{k}_{\rm e^+})\gamma_{\alpha}(1-\gamma^5)\hat{k}_{\nu_{\rm e}}\gamma_{\beta}(1-\gamma^5)\} =$$
$$= 24\,\Bigg(E_{\rm e^+}E_{\nu_{\rm e}} - \frac{1}{3}\vec{k}_{\rm e^+}\cdot \vec{k}_{\nu_{\rm e}}\Bigg),$$
$$\frac{1}{4} \times {\rm tr}\{(M_{\rm N} - \hat{p}_2)\gamma^5 (M_{\rm N} + \hat{p}_1)\gamma^5\} = 2\,M^2_{\rm N}, \quad (3.2)$$

where we have neglected the relative kinetic energy of the protons with respect to the mass of the proton.

Substituting Eq. (3.2) in Eq. (3.1) we get

$$\overline{|{\cal M}({\rm p} + {\rm p} \to {\rm D} + {\rm e}^+ + \nu_e)|^2} = G^2_{\rm V}\,g^2_{\rm A} M^6_{\rm N}\,G^2_{\pi\rm NN}\,\frac{54 Q_{\rm D}}{\pi^2}\,F^2_{\rm D}(k^2)$$
$$\times \frac{C_0^2(k)}{\Big[1 - \frac{1}{2}\,a^{\rm e}_{\rm pp} r^{\rm e}_{\rm pp} k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C)\Big]^2 + (a^{\rm e}_{\rm pp})^2 k^2 C^4_0(k)}\Bigg(E_{\rm e^+}E_{\nu_{\rm e}} - \frac{1}{3}\vec{k}_{\rm e^+}\cdot \vec{k}_{\nu_{\rm e}}\Bigg). \quad (3.3)$$

The integration over the phase volume of the final ${\rm De}^+\nu_{\rm e}$–state we perform in the non–relativistic limit

$$\int \frac{d^3k_{\rm D}}{(2\pi)^3 2E_{\rm D}}\frac{d^3k_{\rm e^+}}{(2\pi)^3 2E_{\rm e^+}}\frac{d^3k_{\nu_{\rm e}}}{(2\pi)^3 2E_{\nu_{\rm e}}}\,(2\pi)^4\,\delta^{(4)}(k_{\rm D} + k_{\ell} - p_1 - p_2)\,\Bigg(E_{\rm e^+}E_{\nu_{\rm e}} - \frac{1}{3}\vec{k}_{\rm e^+}\cdot \vec{k}_{\nu_{\rm e}}\Bigg)$$
$$= \frac{1}{32\pi^3 M_{\rm N}}\int^{W + T_{\rm pp}}_{m_{\rm e}}\sqrt{E^2_{\rm e^+} - m^2_{\rm e}}\,E_{\rm e^+}(W + T_{\rm pp} - E_{\rm e^+})^2\,dE_{\rm e^+} = \frac{(W + T_{\rm pp})^5}{960\pi^3 M_{\rm N}}\,f(\xi), \quad (3.4)$$

where $W = \varepsilon_{\rm D} - (M_{\rm n} - M_{\rm p}) = (2.225 - 1.293)\,{\rm MeV} = 0.932\,{\rm MeV}$ and $\xi = m_{\rm e}/(W + T_{\rm pp})$. The function $f(\xi)$ is defined by the integral

$$f(\xi) = 30\int^1_{\xi}\sqrt{x^2 - \xi^2}\,x\,(1 - x)^2 dx = (1 - \frac{9}{2}\,\xi^2 - 4\,\xi^4)\,\sqrt{1 - \xi^2}$$
$$+ \frac{15}{2}\,\xi^4\,\ell n\Bigg(\frac{1 + \sqrt{1 - \xi^2}}{\xi}\Bigg)\Bigg|_{T_{\rm pp} = 0} = 0.222 \quad (3.5)$$

and normalized to unity at $\xi = 0$.

Thus, the cross section for the solar proton burning is given by

$$\sigma_{\rm pp}(T_{\rm pp}) = \frac{e^{-\pi/r_C\sqrt{M_{\rm N}T_{\rm pp}}}}{v^2}\,\alpha\,\frac{9 g^2_{\rm A} G^2_{\rm V} Q_{\rm D} M^3_{\rm N}}{320\,\pi^4}\,G^2_{\pi\rm NN}\,(W + T_{\rm pp})^5\,f\Bigg(\frac{m_{\rm e}}{W + T_{\rm pp}}\Bigg)$$

$$\times \frac{F^2_{\rm D}(M_{\rm N}T_{\rm pp})}{\Big[1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}M_{\rm N}T_{\rm pp} + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2r_C\sqrt{M_{\rm N}T_{\rm pp}})\Big]^2 + (a^{\rm e}_{\rm pp})^2M_{\rm N}T_{\rm pp}C^4_0(\sqrt{M_{\rm N}T_{\rm pp}})} =$$

$$= \frac{S_{\rm pp}(T_{\rm pp})}{T_{\rm pp}}\,e^{-\pi/r_C\sqrt{M_{\rm N}T_{\rm pp}}}. \tag{3.6}$$

The astrophysical factor $S_{\rm pp}(T_{\rm pp})$ reads

$$S_{\rm pp}(T_{\rm pp}) = \alpha\,\frac{9g^2_{\rm A}G^2_{\rm V}Q_{\rm D}M^4_{\rm N}}{1280\pi^4}\,G^2_{\pi\rm NN}\,(W + T_{\rm pp})^5\,f\Bigg(\frac{m_{\rm e}}{W + T_{\rm pp}}\Bigg)$$
$$\times \frac{F^2_{\rm D}(M_{\rm N}T_{\rm pp})}{\Big[1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}M_{\rm N}T_{\rm pp} + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2r_C\sqrt{M_{\rm N}T_{\rm pp}})\Big]^2 + (a^{\rm e}_{\rm pp})^2M_{\rm N}T_{\rm pp}C^4_0(\sqrt{M_{\rm N}T_{\rm pp}})}. \tag{3.7}$$

At zero kinetic energy of the relative movement of the protons $T_{\rm pp} = 0$ the astrophysical factor $S_{\rm pp}(0)$ is given by

$$S_{\rm pp}(0) = \alpha\,\frac{9g^2_{\rm A}G^2_{\rm V}Q_{\rm D}M^4_{\rm N}}{1280\pi^4}\,G^2_{\pi\rm NN}\,W^5\,f\Bigg(\frac{m_{\rm e}}{W}\Bigg) = 4.08\ \times 10^{-25}\,{\rm MeV\,b}. \tag{3.8}$$

The value $S_{\rm pp}(0) = 4.08\times 10^{-25}\,{\rm MeV\,b}$ agrees good with the recommended value $S_{\rm pp}(0) = 4.00\times 10^{-25}\,{\rm MeV\,b}$ [4]. Insignificant disagreement with the result obtained in Ref.[11] where we have found $S_{\rm pp}(0) = 4.02\times 10^{-25}\,{\rm MeV\,b}$ is due to the new value of the constant $g_{\rm A} = 1.260 \to 1.267$ [18] (see Ref.[13]).

Unlike the astrophysical factor obtained by Kamionkowski and Bahcall [5] the astrophysical factor given by Eq.(3.8) does not depend explicitly on the S–wave scattering wave of pp scattering. This is due to the normalization of the wave function of the relative movement of two protons. After the summation of an infinite series and by using the relation Eq.(2.19) we obtain the wave function of two protons in the form

$$\psi_{\rm pp}(k) = e^{i\,\delta^{\rm e}_{\rm pp}(k)}\,\frac{\sin\,\delta^{\rm e}_{\rm pp}(k)}{-a^{\rm e}_{\rm pp}kC_0(k)}, \tag{3.9}$$

that corresponds the normalization of the wave function of the relative movement of two protons used by Schiavilla *et al.* [6]. For the more detailed discussion of this problem we relegate readers to the paper by Schiavilla *et al.* [6][1].

Unfortunately, the value of the astrophysical factor $S_{\rm pp}(0) = 4.08\times 10^{-25}\,{\rm MeV\,b}$ does not confirm the enhancement by a factor of 1.4 obtained in the modified version of the RFMD in Ref.[14].

4 Neutrino disintegration of the deuteron induced by charged weak current

The evaluation of the amplitude of the process $\nu_{\rm e} + {\rm D} \to {\rm e}^- + {\rm p} + {\rm p}$ has been given in details in Ref. [10]. The result can be written in the following form

$$i{\cal M}({\rm p + p \to D + e^+} + \nu_e) = g_{\rm A}M_{\rm N}\frac{G_{\rm V}}{\sqrt{2}}\,\frac{3g_{\rm V}}{2\pi^2}\,G_{\pi\rm NN}\,e^*_{\mu}(k_{\rm D})\,[\bar{u}(k_{\rm e^-})\gamma^{\mu}(1-\gamma^5)u(k_{\nu_{\rm e}})]\,{\cal F}^{\rm e}_{\rm ppe^-}$$

[1]See the last paragraph of Sect. 3 and the first paragraph of Sect. 5 of Ref.[6].

$$\times \frac{C_0(k)}{1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C) + i\,a^{\rm e}_{\rm pp}\,k\,C^2_0(k)}\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,F_{\rm D}(k^2), \qquad (4.1)$$

where $\mathcal{F}^{\rm e}_{\rm ppe^-}$ is the overlap factor which we evaluate below, and $F_{\rm D}(k^2)$ is the universal form factor Eq.(2.17) describing a spatial smearing of the deuteron [14].

The amplitude Eq.(4.1) squared, averaged over polarizations of the deuteron and summed over polarizations of the final particles reads

$$\overline{|\mathcal{M}(\nu_{\rm e} + {\rm D} \to {\rm e}^- + {\rm p} + {\rm p})|^2} = g^2_{\rm A}M^6_{\rm N}\frac{144G^2_{\rm V}Q_{\rm D}}{\pi^2}\,G^2_{\pi{\rm NN}}\,|\mathcal{F}^{\rm e}_{\rm ppe^-}|^2\;F^2_{\rm D}(k^2)\,F(Z,E_{\rm e^-})$$
$$\times\frac{C^2_0(k)}{\Big[1 - \frac{1}{2}a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_{\rm C}}\,h(2kr_{\rm C})\Big]^2 + (a^{\rm e}_{\rm pp})^2k^2C^4_0(k)}\Big(E_{\rm e^-}E_{\nu_{\rm e}} - \frac{1}{3}\vec{k}_{\rm e^-}\cdot\vec{k}_{\nu_{\rm e}}\Big), \qquad (4.2)$$

where $F(Z, E_{\rm e^-}$ is the Fermi function [21] describing the Coulomb interaction of the electron with the nuclear system having a charge Z. In the case of the reaction $\nu_{\rm e}$ + D $\to$ e$^+$ + p + p we have $Z = 2$. At $\alpha^2Z^2 \ll 1$ the Fermi function $F(Z, E_{\rm e^-})$ reads [21]

$$F(Z, E_{\rm e^-}) = \frac{2\pi\eta_{\rm e^-}}{1 - e^{-2\pi\eta_{\rm e^-}}}, \qquad (4.3)$$

where $\eta_{\rm e^-} = Z\alpha/v_{\rm e^-} = Z\alpha E_{\rm e^-}/\sqrt{E^2_{\rm e^-} - m^2_{\rm e^-}}$ and $v_{\rm e^-}$ is a velocity of the electron.

The r.h.s. of Eq.(4.2) can be expressed in terms of the astrophysical factor $S_{\rm pp}(0)$ for the solar proton burning brought up to the form

$$\overline{|\mathcal{M}(\nu_{\rm e} + {\rm D} \to {\rm e}^- + {\rm p} + {\rm p})|^2} = S_{\rm pp}(0)\,\frac{2^{12}5\pi^2}{\Omega_{{\rm De}^+\nu_{\rm e}}}\,\frac{r_{\rm C}M^3_{\rm N}}{m^5_{\rm e}}\,\frac{|\mathcal{F}^{\rm e}_{\rm ppe^-}|^2}{|\mathcal{F}^{\rm e}_{\rm pp}|^2}\,F^2_{\rm D}(k^2)\,F(Z,E_{\rm e^-})$$
$$\times\frac{C^2_0(k)}{\Big[1 - \frac{1}{2}a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_{\rm C}}\,h(2kr_{\rm C})\Big]^2 + (a^{\rm e}_{\rm pp})^2k^2C^4_0(k)}\Big(E_{\rm e^-}E_{\nu_{\rm e}} - \frac{1}{3}\vec{k}_{\rm e^-}\cdot\vec{k}_{\nu_{\rm e}}\Big). \qquad (4.4)$$

We have used here the expression for the astrophysical factor

$$S_{\rm pp}(0) = \frac{9g^2_{\rm A}G^2_{\rm V}Q_{\rm D}M^3_{\rm N}}{1280\pi^4r_{\rm C}}\,G^2_{\pi{\rm NN}}\,|\mathcal{F}^{\rm e}_{\rm pp}|^2\,m^5_{\rm e}\,\Omega_{{\rm De}^+\nu_{\rm e}}, \qquad (4.5)$$

where $m_{\rm e} = 0.511\,{\rm MeV}$ is the electron mass, and $\Omega_{{\rm De}^+\nu_{\rm e}} = (W/m_{\rm e})^5f(m_{\rm e}/W) = 4.481$ at $W = 0.932\,{\rm MeV}$. The function $f(m_{\rm e}/W)$ is defined by Eq.(3.5).

In the rest frame of the deuteron the cross section for the process $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p is defined as

$$\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}}) = \frac{1}{4M_{\rm D}E_{\nu_{\rm e}}}\int\overline{|\mathcal{M}(\nu_{\rm e} + {\rm D} \to {\rm e}^- + {\rm p} + {\rm p})|^2}$$
$$\frac{1}{2}\,(2\pi)^4\,\delta^{(4)}(k_{\rm D} + k_{\nu_{\rm e}} - p_1 - p_2 - k_{\rm e^-})\,\frac{d^3p_1}{(2\pi)^32E_1}\frac{d^3p_2}{(2\pi)^32E_2}\frac{d^3k_{\rm e^-}}{(2\pi)^32E_{\rm e^-}}, \qquad (4.6)$$

where $E_{\nu_{\rm e}}$, E_1, E_2 and $E_{\rm e^-}$ are the energies of the neutrino, the protons and the electron. The abbreviation (cc) means the charged current. The integration over the phase volume

of the (ppe$^-$)–state we perform in the non–relativistic limit and in the rest frame of the deuteron,

$$
\begin{aligned}
&\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3k_{\rm e}}{(2\pi)^3 2E_{\rm e^-}}(2\pi)^4\,\delta^{(4)}(k_{\rm D}+k_{\nu_{\rm e}}-p_1-p_2-k_{\rm e^-})\\
&\frac{C^2_0(\sqrt{M_{\rm N}T_{\rm pp}})\,F^2_{\rm D}(M_{\rm N}T_{\rm pp})\,F(Z,E_{\rm e^-})}{\Big[1-\dfrac{1}{2}a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}M_{\rm N}T_{\rm pp}+\dfrac{a^{\rm e}_{\rm pp}}{r_{\rm C}}\,h(2r_{\rm C}\sqrt{M_{\rm N}T_{\rm pp}})\Big]^2+(a^{\rm e}_{\rm pp})^2M_{\rm N}T_{\rm pp}C^4_0(\sqrt{M_{\rm N}T_{\rm pp}})}\\
&\left(E_{\rm e^-}E_{\nu_{\rm e}}-\frac{1}{3}\vec{k}_{\rm e^-}\cdot\vec{k}_{\nu_{\rm e}}\right)=\frac{E_{\bar{\nu}_{\rm e}}M^3_{\rm N}}{128\pi^3}\left(\frac{E_{\rm th}}{M_{\rm N}}\right)^{7/2}\left(\frac{2m_{\rm e}}{E_{\rm th}}\right)^{3/2}\frac{1}{E^2_{\rm th}}\\
&\iint dT_{\rm e^-}dT_{\rm pp}\delta(E_{\nu_{\rm e}}-E_{\rm th}-T_{\rm e^-}-T_{\rm pp})\sqrt{T_{\rm e^-}T_{\rm pp}}\left(1+\frac{T_{\rm e^-}}{m_{\rm e}}\right)\sqrt{1+\frac{T_{\rm e^-}}{2m_{\rm e}}}\\
&\frac{C^2_0(\sqrt{M_{\rm N}T_{\rm pp}})\,F^2_{\rm D}(M_{\rm N}T_{\rm pp})\,F(Z,E_{\rm e^-})}{\Big[1-\dfrac{1}{2}a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}M_{\rm N}T_{\rm pp}+\dfrac{a^{\rm e}_{\rm pp}}{r_{\rm C}}\,h(2r_{\rm C}\sqrt{M_{\rm N}T_{\rm pp}})\Big]^2+(a^{\rm e}_{\rm pp})^2M_{\rm N}T_{\rm pp}C^4_0(\sqrt{M_{\rm N}T_{\rm pp}})}\\
&=\frac{E_{\nu_{\rm e}}M^3_{\rm N}}{128\pi^3}\left(\frac{E_{\rm th}}{M_{\rm N}}\right)^{7/2}\left(\frac{2m_{\rm e}}{E_{\rm th}}\right)^{3/2}(y-1)^2\,\Omega_{\rm ppe^-}(y), \qquad (4.7)
\end{aligned}
$$

where $T_{\rm e^-}$ is the kinetic energy of the electron, $E_{\rm th}$ is the neutrino energy threshold of the reaction $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p, and is given by $E_{\rm th}=\varepsilon_{\rm D}+m_{\rm e}-(M_{\rm n}-M_{\rm p})=(2.225+0.511-1.293)\,{\rm MeV}=1.443\,{\rm MeV}$. The function $\Omega_{\rm ppe^-}(y)$, where $y=E_{\nu_{\rm e}}/E_{\rm th}$, is defined as

$$
\begin{aligned}
\Omega_{\rm ppe^-}(y)=&\int^1_0 dx\sqrt{x(1-x)}\left(1+\frac{E_{\rm th}}{m_{\rm e}}(y-1)(1-x)\right)\sqrt{1+\frac{E_{\rm th}}{2m_{\rm e}}(y-1)(1-x)}\\
&C^2_0(\sqrt{M_{\rm N}E_{\rm th}\,(y-1)\,x})\,F^2_{\rm D}(M_{\rm N}E_{\rm th}\,(y-1)\,x)\,F(Z,m_{\rm e}+E_{\rm th}(y-1)\,(1-x))\\
&\Big\{\Big[1-\frac{1}{2}a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}M_{\rm N}E_{\rm th}\,(y-1)\,x+\frac{a^{\rm e}_{\rm pp}}{r_{\rm C}}\,h(2r_{\rm C}\sqrt{M_{\rm N}E_{\rm th}\,(y-1)\,x})\Big]^2\\
&\quad+(a^{\rm e}_{\rm pp})^2\,M_{\rm N}E_{\rm th}\,(y-1)\,xC^4_0(\sqrt{M_{\rm N}E_{\rm th}\,(y-1)\,x})\Big\}^{-1}, \qquad (4.8)
\end{aligned}
$$

where we have changed the variable $T_{\rm pp}=(E_{\nu_{\rm e}}-E_{\rm th})\,x$.

The cross section for $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p is defined

$$
\begin{aligned}
\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}}) &= S_{\rm pp}(0)\,\frac{640r_{\rm C}}{\pi\Omega_{{\rm De}^+\nu_{\rm e}}}\left(\frac{M_{\rm N}}{E_{\rm th}}\right)^{3/2}\left(\frac{E_{\rm th}}{2m_{\rm e}}\right)^{7/2}\frac{|{\cal F}^{\rm e}_{\rm ppe^-}|^2}{|{\cal F}^{\rm e}_{\rm pp}|^2}\,(y-1)^2\,\Omega_{\rm ppe^-}(y)=\\
&= 3.69\times 10^5\,S_{\rm pp}(0)\,\frac{|{\cal F}^{\rm e}_{\rm ppe^-}|^2}{|{\cal F}^{\rm e}_{\rm pp}|^2}\,(y-1)^2\,\Omega_{\rm ppe^-}(y), \qquad (4.9)
\end{aligned}
$$

where $S_{\rm pp}(0)$ is measured in MeV cm^2. For $S_{\rm pp}(0)=4.08\times 10^{-49}\,{\rm MeV\,cm^2}$ Eq.(3.8) the cross section $\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}})$ reads

$$
\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}})=1.50\,\frac{|{\cal F}^{\rm e}_{\rm ppe^-}|^2}{|{\cal F}^{\rm e}_{\rm pp}|^2}\,(y-1)^2\,\Omega_{\rm ppe^-}(y)\,10^{-43}\,{\rm cm}^2. \qquad (4.10)
$$

In order to make numerical predictions for the cross section Eq.(4.10) we should evaluate the overlap factor $\mathcal{F}^{\rm e}_{\rm ppe^-}$. This evaluation can be carried out in analogy with the evaluation of $\mathcal{F}^{\rm e}_{\rm pp}$. By using the results obtained in Ref.[10] we get

$$\mathcal{F}^{\rm e}_{\rm ppe^-} = 1 + \frac{32}{9}\int\limits_0^{\infty} dv\, v^2\,\Big[e^{-\alpha\pi/4v}\,\Gamma\Big(1 - \frac{i\alpha}{2v}\Big) - 1\Big]\Big[\frac{1}{(1+v^2)^{5/2}} - \frac{1}{16}\,\frac{1}{(1+v^2)^{3/2}}\Big]. \quad (4.11)$$

The perturbative evaluation of the integral gives

$$\mathcal{F}^{\rm e}_{\rm ppe^-} = 1 - \alpha\left(\frac{13\pi}{54} - i\,\frac{13\gamma}{27}\right) + O(\alpha^2). \quad (4.12)$$

Thus, the overlap factor $\mathcal{F}^{\rm e}_{\rm ppe^-}$ differs slightly from unity as well as the overlap factor $\mathcal{F}^{\rm e}_{\rm pp}$ of the solar proton burning. The ratio of the overlap factors is equal to

$$\frac{|\mathcal{F}^{\rm e}_{\rm ppe^-}|^2}{|\mathcal{F}^{\rm e}_{\rm pp}|^2} = 1 - \alpha\,\frac{2\pi}{3} + O(\alpha^2) = 1 + (-1.53\times 10^{-2}) \simeq 1. \quad (4.13)$$

Setting $|\mathcal{F}^{\rm e}_{\rm ppe^-}|^2/|\mathcal{F}^{\rm e}_{\rm pp}|^2 = 1$ we can make numerical predictions for the cross section Eq.(4.10) and compare them with the PMA ones.

The most recent PMA calculations the cross section for the reaction $\nu_{\rm e} + {\rm D} \to {\rm e}^- + {\rm p} + {\rm p}$ have been obtained in Refs. [22,23] and tabulated for the neutrino energies ranging over the region from threshold up to 160 MeV. Since our result is restricted by the neutrino energies from threshold up to 10 MeV, we compute the cross section only for this energy region

$$\begin{aligned}
\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}} = 4\,{\rm MeV}) &= 2.46\,(1.86/1.54)\times 10^{-43}\,{\rm cm}^2,\\
\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}} = 6\,{\rm MeV}) &= 9.60\,(5.89/6.13)\times 10^{-43}\,{\rm cm}^2,\\
\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}} = 8\,{\rm MeV}) &= 2.38\,(1.38/1.44)\times 10^{-42}\,{\rm cm}^2,\\
\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}} = 10\,{\rm MeV}) &= 4.07\,(2.55/2.66)\times 10^{-43}\,{\rm cm}^2,
\end{aligned} \quad (4.14)$$

where the data in parentheses are taken from Refs. [22] and [23], respectively. Thus, on the average our numerical values for the cross section $\sigma^{\nu_{\rm e}D}_{\rm cc}(E_{\nu_{\rm e}})$ by a factor of 1.5 are larger compared with the PMA ones.

Our predictions for the cross section Eq.(4.14) differ from the predictions of Ref.[14]. This is related to (i) the value of the astrophysical factor which is by a factor 1.4 larger in Ref.[14] and (ii) the form factor describing a spatial smearing of the deuteron which is $F^2_{\rm D}(k^2)$ is this paper (see Ref. [13]) and $F_{\rm D}(k^2)$ in Ref.[14].

5 Astrophysical factor for pep process

In the RFMD the amplitude of the reaction ${\rm p} + {\rm e}^- + {\rm p} \to {\rm D} + \nu_{\rm e}$ or the pep–process is related to the effective Lagrangian Eq.(2.3) and reads

$$i\mathcal{M}({\rm p} + {\rm e}^- + {\rm p} \to {\rm D} + \nu_e) = G_{\rm V}\,g_{\rm A}M_{\rm N}\,G_{\pi{\rm NN}}\,\frac{3g_{\rm V}}{4\pi^2}\,e^*_{\mu}(k_{\rm D})\,[\bar{u}(k_{\nu_{\rm e}})\gamma^{\mu}(1-\gamma^5)u(k_{{\rm e}^-})]\,\mathcal{F}^{\rm e}_{\rm pp}$$
$$\times\,\frac{C_0(k)}{1 - \frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2 + \frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C) + i\,a^{\rm e}_{\rm pp}\,k\,C^2_0(k)}\,[\bar{u^c}(p_2)\gamma^5u(p_1)]\,F_{\rm D}(k^2), \quad (5.1)$$

where we have described low–energy elastic pp scattering in analogy with the solar proton burning and the neutrino disintegration of the deuteron.

The amplitude Eq.(5.1) squared, averaged and summed over polarizations of the interacting particles is defined

$$\overline{|\mathcal{M}(\mathrm{p}+\mathrm{e}^-+\mathrm{p}\to\mathrm{D}+\nu_e)|^2} = G^2_{\rm V}\,g^2_{\rm A}M^6_{\rm N}\,G^2_{\pi\rm NN}\,\frac{27Q_{\rm D}}{\pi^2}\,|\mathcal{F}^{\rm e}_{\rm pp}|^2\;F^2_{\rm D}(k^2)\,F(Z,E_{\rm e^-})$$
$$\times\,\frac{C^2_0(k)}{\Big[1-\frac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2+\frac{a^{\rm e}_{\rm pp}}{r_C}\,h(2kr_C)\Big]^2+(a^{\rm e}_{\rm pp})^2k^2C^4_0(k)}\,\Big(E_{\rm e^+}E_{\nu_{\rm e}}-\frac{1}{3}\vec{k}_{\rm e^+}\cdot\vec{k}_{\nu_{\rm e}}\Big), \quad (5.2)$$

where $F(Z,E_{\rm e^-})$ is the Fermi function given by Eq.(4.3).

At low energies the cross section $\sigma_{\rm pep}(T_{\rm pp})$ for the pep–process can be determined as follows [24]

$$\sigma_{\rm pep}(T_{\rm pp}) = \frac{1}{v}\frac{1}{4M^2_{\rm N}}\int\frac{d^3k_{\rm e^-}}{(2\pi)^3 2E_{\rm e^-}}\,g\,n(\vec{k}_{\rm e^-})\int\overline{|\mathcal{M}(\mathrm{p}+\mathrm{e}^-+\mathrm{p}\to\mathrm{D}+\nu_{\rm e})|^2}$$
$$(2\pi)^4\delta^{(4)}(k_{\rm D}+k_{\nu_{\rm e}}-p_1-p_2-k_{\rm e^-})\frac{d^3k_{\rm D}}{(2\pi)^3 2M_{\rm D}}\frac{d^3k_{\nu_{\rm e}}}{(2\pi)^3 2E_{\nu_{\rm e}}}, \quad (5.3)$$

where $g=2$ is the number of the electron spin states and v is a relative velocity of the protons. The electron distribution function $n(\vec{k}_{\rm e^-})$ can be taken in the form [21]

$$n(\vec{k}_{\rm e^-}) = e^{\bar{\nu}\,-\,T_{\rm e^-}/kT_c}, \quad (5.4)$$

where $k = 8.617\times 10^{-11}\,{\rm MeV\,K^{-1}}$, T_c is a temperature of the core of the Sun. The distribution function $n(\vec{k}_{\rm e^-})$ is normalized by the condition

$$g\int\frac{d^3k_{\rm e^-}}{(2\pi)^3}\,n(\vec{k}_{\rm e^-}) = n_{\rm e^-}, \quad (5.5)$$

where $n_{\rm e^-}$ is the electron number density. From the normalization condition Eq.(5.5) we derive

$$e^{\bar{\nu}} = \frac{4\,\pi^3\,n_{\rm e^-}}{(2\pi\,m_{\rm e}\,kT_c)^{3/2}}. \quad (5.6)$$

The astrophysical factor $S_{\rm pep}(0)$ is then defined by

$$S_{\rm pep}(0) = S_{\rm pp}(0)\,\frac{15}{2\pi}\,\frac{1}{\Omega_{\rm De^+\nu_e}}\,\frac{1}{m^3_{\rm e}}\Big(\frac{E_{\rm th}}{m_{\rm e}}\Big)^2\,e^{\bar{\nu}}\,\int d^3k_{\rm e^-}\,e^{-T_{\rm e^-}/kT_c}\,F(Z,E_{\rm e^-}). \quad (5.7)$$

For the ratio $S_{\rm pep}(0)/S_{\rm pp}(0)$ we obtain

$$\frac{S_{\rm pep}(0)}{S_{\rm pp}(0)} = \frac{2^{3/2}\pi^{5/2}}{f_{\rm pp}(0)}\Big(\frac{\alpha Z n_{\rm e^-}}{m^3_{\rm e}}\Big)\Big(\frac{E_{\rm th}}{m_{\rm e}}\Big)^2\sqrt{\frac{m_{\rm e}}{kT_c}}\,I\Big(Z\sqrt{\frac{2m_{\rm e}}{kT_c}}\Big). \quad (5.8)$$

We have set $f_{\rm pp}(0) = \Omega_{\rm De^+\nu_e}/30 = 0.149$ [21] and the function $I(x)$ having been introduced by Bahcall and May [21] reads

$$I(x) = \int\limits^{\infty}_{0}\frac{du\,e^{-u}}{1-e^{-\pi\alpha\,x/\sqrt{u}}}. \quad (5.9)$$

The relation between the astrophysical factors $S_{\rm pep}(0)$ and $S_{\rm pp}(0)$ given by Eq.(5.8) is in complete agreement with that obtained by Bahcall and May [21]. The ratio Eq.(5.8) does not depend on whether the astrophysical factor $S_{\rm pp}(0)$ is enhanced with respect to the recommended value or not.

6 Conclusion

We have shown that the contributions of low–energy elastic pp scattering in the ${}^1{\rm S}_0$–state with the Coulomb repulsion to the amplitudes of the reactions p + p → D + e^+ + $\nu_{\rm e}$, $\nu_{\rm e}$ + D → e^- + p + p and p + e^- + p → D + $\nu_{\rm e}$ can be described in the RFMD in full agreement with low–energy nuclear phenomenology in terms of the S–wave scattering length and the effective range. The amplitude of low–energy elastic pp scattering has been obtained by summing up an infinite series of one–proton loop diagrams and the evaluation of the result of the summation in leading order in the large N_C expansion. This takes away fully the problem pointed out by Bahcall and Kamionkowski [17] that in the RFMD with the effective local four–nucleon interaction Eq.(2.1) one cannot describe low–energy elastic pp scattering in the ${}^1{\rm S}_0$–state with the Coulomb repulsion in agreement with low–energy nuclear phenomenology.

The obtained numerical value of the astrophysical factor $S_{\rm pp}(0) = 4.08\times 10^{-25}\,{\rm MeV\,b}$ agrees with the recommended value $S_{\rm pp}(0) = 4.00\times 10^{-25}\,{\rm MeV\,b}$ and recent estimate $S_{\rm pp}(0) = 4.20\times 10^{-25}\,{\rm MeV\,b}$ [9] obtained from the helioseismic data.

Unfortunately, the value of the astrophysical factor $S_{\rm pp}(0) = 4.08\times 10^{-25}\,{\rm MeV\,b}$ does not confirm the enhancement by a factor of 1.4 obtained in the modified version of the RFMD in Ref.[14] which is not well defined due to a violation of Lorentz invariance of the effective four–nucleon interaction describing N + N → N + N transitions. This violation has turned out to be incompatible with a dominance of one–nucleon loop anomalies which are Lorentz covariant.

The cross section for the neutrino disintegration of the deuteron has been evaluated with respect to $S_{\rm pp}(0)$. We have obtained an enhancement of the cross section by a factor of order 1.5 on the average for neutrino energies $E_{\nu_{\rm e}}$ varying from threshold to $E_{\nu_{\rm e}} \le 10\,{\rm MeV}$. It would be important to verify our results for the reaction $\nu_{\rm e}$ + D → e^- + p + p in solar neutrino experiments planned by SNO. In fact, first, this should provide an experimental study of $S_{\rm pp}(0)$ and, second, the cross sections for the anti–neutrino disintegration of the deuteron caused by charged $\bar{\nu}_{\rm e}$ + D → e^+ + n + n and neutral $\bar{\nu}_{\rm e}$ + D → $\bar{\nu}_{\rm e}$ + n + p weak currents have been found in good agreement with recent experimental data obtained by the Reines's experimental group [26].

The evaluation of the astrophysical factor $S_{\rm pep}(0)$ for the reaction p + e^- + p → D + $\nu_{\rm e}$ or pep–process in the RFMD has shown that the ratio $S_{\rm pep}(0)/S_{\rm pp}(0)$, first, agrees fully with the result obtained by Bahcall and May [21] and, second, does not depend on whether $S_{\rm pp}(0)$ is enhanced with respect to the recommended value or not.

Concluding the paper we would like to emphasize that our model, the RFMD, conveys the idea of a dominant role of one–fermion loop (one–nucleon loop) anomalies from elementary particle physics to the nuclear one. This is a new approach to the description of low–energy nuclear forces in physics of finite nuclei. In spite of almost 30 year's history after the discovery of one–fermion loop anomalies and application of these anomalies to

the evaluation of effective Lagrangians of low–energy interactions of hadrons, in nuclear physics fermion–loop anomalies have not been applied to the analysis of low–energy nuclear interactions and properties of nuclei. However, an important role of $N\bar{N}$ fluctuations for the correct description of low–energy properties of finite nuclei has been understood in Ref.[16]. Moreover, $N\bar{N}$ fluctuations have been described in terms of one–nucleon loop diagrams within quantum field theoretic approaches, but the contributions of one–nucleon loop anomalies have not been considered in the papers of Ref.[16].

The RFMD strives to fill this blank. Within the framework of the RFMD we aim to understand, in principle, the possibility of the description of strong low-energy nuclear forces in terms of one–nucleon loop anomalies. Of course, our results should be quantitatively compared with the experimental data and other theoretical approaches. Nevertheless, at the present level of the development of our model one cannot demand at once to describe, for example, the astrophysical factor $S_{\rm pp}(0)$ with accuracy better than it has been carried out by Schiavilla *et al.* [6], where only corrections not greater than 1% are allowed. It is not important for our approach at present. What is much more important is in the possibility to describe without free parameters in quantitative agreement with both the experimental data and other theoretical approaches all multitude of low–energy nuclear reactions of the deuteron coupled to nucleons and other particles. In Ref.[13] we have outlined the procedure of the evaluation of chiral meson–loop corrections in the RFMD. The absence of free parameters in the RFMD gives the possibility to value not only the role of these corrections but also the corrections of other kind mentioned recently by Vogel and Beacom [25].

The justification of the RFMD within QCD and large N_C expansion [12] implies that one–nucleon loop anomalies might be natural objects for the understanding of low-energy nuclear forces. The real accuracy of the approach should be found out for the process of the development.

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